

Contents

<i>Preface</i>	vi
<i>Introduction</i>	1
1 The concept of nonuniform hyperbolicity	6
1.1 Motivation	6
1.2 Basic setting	10
1.2.1 Exponential splitting and nonuniform hyperbolicity	10
1.2.2 Tempered equivalence	11
1.2.3 The continuous-time case	12
1.3 Lyapunov exponents associated to sequences of matrices	13
1.3.1 Definition of the Lyapunov exponent	13
1.3.2 Forward and backward regularity	15
1.3.3 A criterion of forward regularity for triangular matrices	23
1.3.4 The Lyapunov–Perron regularity	29
1.4 Notes	31
2 Lyapunov exponents for linear extensions	33
2.1 Cocycles over dynamical systems	33
2.1.1 Cocycles and linear extensions	33
2.1.2 Cohomology and tempered equivalence	35
2.1.3 Examples and basic constructions	38
2.2 Hyperbolicity of cocycles	39
2.2.1 Hyperbolic cocycles	39
2.2.2 Regular sets of hyperbolic cocycles	42
2.2.3 Cocycles over topological spaces	44
2.3 Lyapunov exponents for cocycles	45
2.4 Spaces of cocycles	49
3 Regularity of cocycles	51
3.1 The Lyapunov–Perron regularity	51
3.2 Lyapunov exponents and basic constructions	55

3.3	Lyapunov exponents and hyperbolicity	57
3.4	The Multiplicative Ergodic Theorem	62
3.4.1	One-dimensional cocycles and Birkhoff's Ergodic Theorem	63
3.4.2	Oseledets' proof of the Multiplicative Ergodic Theorem	63
3.4.3	Lyapunov exponents and Sub-Additive Ergodic Theorem	68
3.4.4	Raghunathan's proof of the Multiplicative Ergodic Theorem	69
3.5	Tempering kernels and the Reduction Theorems	74
3.5.1	Lyapunov inner products	75
3.5.2	The Oseledets–Pesin Reduction Theorem	76
3.5.3	A tempering kernel	79
3.5.4	Zimmer's amenable reduction	81
3.5.5	The case of noninvertible cocycles	81
3.6	More results on the Lyapunov–Perron regularity	82
3.6.1	Higher-rank Abelian actions	82
3.6.2	The case of flows	87
3.6.3	Nonpositively curved spaces	91
3.7	Notes	93
4	Methods for estimating exponents	95
4.1	Cone and Lyapunov function techniques	95
4.1.1	Lyapunov functions	96
4.1.2	A criterion for nonvanishing Lyapunov exponents	98
4.1.3	Invariant cone families	101
4.2	Cocycles with values in the symplectic group	102
4.3	Monotone operators and Lyapunov exponents	106
4.3.1	The algebra of Potapov	106
4.3.2	Lyapunov exponents for \mathcal{J} -separated cocycles	108
4.3.3	The Lyapunov spectrum for conformally Hamiltonian systems	113
4.4	A remark on applications of cone techniques	117
4.5	Notes	118
5	The derivative cocycle	119
5.1	Smooth dynamical systems and the derivative cocycle	119
5.2	Nonuniformly hyperbolic diffeomorphisms	120
5.3	Hölder continuity of invariant distributions	123
5.4	Lyapunov exponent and regularity of the derivative cocycle	127
5.5	On the notion of dynamical systems with nonzero exponents	130
5.6	Regular neighborhoods	131
5.7	Cocycles over smooth flows	134
5.8	Semicontinuity of Lyapunov exponents	136

6	Examples of systems with hyperbolic behavior	138
6.1	Uniformly hyperbolic sets	138
6.1.1	Hyperbolic sets for maps	138
6.1.2	Hyperbolic sets for flows	142
6.1.3	Linear horseshoes	144
6.1.4	Nonlinear horseshoes	146
6.2	Nonuniformly hyperbolic perturbations of horseshoes	152
6.2.1	Slow expansion near a fixed point	152
6.2.2	Further modifications	154
6.3	Diffeomorphisms with nonzero exponents on surfaces	158
6.3.1	Nonuniformly hyperbolic diffeomorphisms of the torus	158
6.3.2	A nonuniformly hyperbolic diffeomorphism on the sphere	163
6.3.3	Nonuniformly hyperbolic diffeomorphisms on compact surfaces	164
6.3.4	Analytic diffeomorphisms	166
6.4	Pseudo-Anosov maps	167
6.4.1	Definitions and basic properties	167
6.4.2	Smooth models of pseudo-Anosov maps	171
6.5	Nonuniformly hyperbolic flows	182
6.6	Some other examples	185
6.7	Notes	188
7	Stable manifold theory	189
7.1	The Stable Manifold Theorem	189
7.2	Nonuniformly hyperbolic sequences of diffeomorphisms	192
7.3	The Hadamard–Perron Theorem: Hadamard’s method	193
7.3.1	Invariant cone families	194
7.3.2	Admissible manifolds	198
7.3.3	Existence of (s, γ) - and (u, γ) -manifolds	201
7.3.4	Invariant families of local manifolds	204
7.3.5	Higher differentiability of invariant manifolds	207
7.4	The Graph Transform Property	208
7.5	The Hadamard–Perron Theorem: Perron’s method	208
7.5.1	An Abstract Version of the Stable Manifold Theorem	209
7.5.2	Smoothness of local manifolds	218
7.6	Local unstable manifolds	223
7.7	The Stable Manifold Theorem for flows	223
7.8	C^1 pathological behavior: Pugh’s example	224
7.9	Notes	227
8	Basic properties of stable and unstable manifolds	229
8.1	Characterization and sizes of local stable manifolds	229

8.2	Global stable and unstable manifolds	232
8.3	Foliations with smooth leaves	234
8.4	Filtrations of intermediate local and global manifolds	235
8.5	The Lipschitz property of intermediate foliations	239
8.6	The absolute continuity property	244
8.6.1	Absolute continuity of holonomy maps	245
8.6.2	Absolute continuity of local stable manifolds	255
8.6.3	Foliation that is not absolutely continuous	258
8.6.4	The Jacobian of the holonomy map	259
8.7	Notes	261
9	Smooth measures	262
9.1	Ergodic components	262
9.2	Local ergodicity	267
9.3	The s - and u -measures	284
9.4	The leaf-subordinated partition and the K -property	287
9.5	The Bernoulli property	292
9.6	The continuous-time case	300
9.7	Notes	305
10	Measure-Theoretic Entropy and Lyapunov exponents	307
10.1	Entropy of measurable transformations	307
10.2	The Margulis–Ruelle inequality	308
10.3	The topological entropy and Lyapunov exponents	311
10.4	The entropy formula	315
10.5	Mañé’s proof of the entropy formula	318
10.6	Notes	328
11	Stable ergodicity and Lyapunov exponents	329
11.1	Uniform partial hyperbolicity and stable ergodicity	329
11.2	Partially hyperbolic systems with nonzero exponents	332
11.3	Hyperbolic diffeomorphisms with countably many ergodic components	340
11.4	Existence of hyperbolic diffeomorphisms on manifolds	352
11.5	Existence of hyperbolic flows on manifolds	374
11.6	Foliations that are not absolutely continuous	382
11.7	Open sets of diffeomorphisms with nonzero exponents	387
11.8	Notes	388
12	Geodesic flows	389
12.1	Hyperbolicity of geodesic flows	389
12.2	Ergodic properties of geodesic flows	398
12.3	Entropy of geodesic flows	409
12.4	Topological properties of geodesic flows	412

12.5	The Teichmüller geodesic flow	414
12.6	Notes	420
13	SRB measures	422
13.1	Definition and ergodic properties of SRB measures	422
13.2	The characterization of SRB measures	426
13.3	Constructions of SRB measures	428
13.4	Notes	435
14	Hyperbolic measures: entropy and dimension	436
14.1	Pointwise dimension and the Ledrappier–Young entropy formula	436
14.1.1	Local entropies	437
14.1.2	Leaf pointwise dimensions	441
14.1.3	The Ledrappier–Young entropy formula	453
14.2	Local product structure of hyperbolic measures	454
14.3	Applications to dimension theory	465
14.4	Notes	466
15	Hyperbolic measures: topological properties	467
15.1	Closing lemma	467
15.2	Shadowing lemma	476
15.3	The Livshitz theorem	478
15.4	Hyperbolic periodic orbits	479
15.5	Topological transitivity and spectral decomposition	487
15.6	Entropy, horseshoes, and periodic points	487
15.7	Continuity properties of entropy	490
	<i>Bibliography</i>	497