Contents

Pref	ace
1 101	

vii

Part 1. The Core of the Theory

Chapter	1. Examples of Hyperbolic Dynamical Systems	3
§1.1.	Anosov diffeomorphisms	4
§1.2.	Anosov flows	9
§1.3.	Hyperbolic sets	14
§1.4.	The Smale–Williams solenoid	20
§1.5.	The Katok map of the 2-torus	22
§1.6.	Area preserving diffeomorphisms with nonzero Lyapunov exponents on surfaces	33
§1.7.	A volume preserving flow with nonzero Lyapunov exponents	39
§1.8.	A slow-down of the Smale–Williams solenoid	43
Chapter	2. General Theory of Lyapunov Exponents	45
$\S{2.1.}$	Lyapunov exponents and their basic properties	45
§2.2.	The Lyapunov and Perron irregularity coefficients	50
§2.3.	Lyapunov exponents for linear differential equations	54
$\S2.4.$	Forward and backward regularity. The Lyapunov–Perron	
	regularity	67
$\S2.5.$	Lyapunov exponents for sequences of matrices	73
Chapter	3. Cocycles over Dynamical Systems	81
§3.1.	Cocycles and linear extensions	82
		iii

§3.2.	Lyapunov exponents and Lyapunov–Perron regularity for cocycles	87
§3.3.	Examples of measurable cocycles over dynamical systems	92
Chapter	4. The Multiplicative Ergodic Theorem	97
§4.1.	Lyapunov–Perron regularity for sequences of triangular	
	matrices	98
$\S4.2.$	Proof of the Multiplicative Ergodic Theorem	104
§4.3.	Normal forms of cocycles	109
§4.4.	Regular neighborhoods	114
Chapter	5. Elements of the Nonuniform Hyperbolicity Theory	119
$\S{5.1.}$	Dynamical systems with nonzero Lyapunov exponents	120
$\S{5.2.}$	Nonuniform complete hyperbolicity	129
$\S{5.3.}$	Regular sets	132
$\S{5.4.}$	Nonuniform partial hyperbolicity	139
$\S{5.5.}$	Hölder continuity of invariant distributions	141
Chapter	6. Lyapunov Stability Theory of Nonautonomous Equations	147
$\S6.1.$	Stability of solutions of ordinary differential equations	148
$\S6.2.$	Lyapunov absolute stability theorem	153
$\S6.3.$	Lyapunov conditional stability theorem	158
Chapter	7. Local Manifold Theory	161
§7.1.	Local stable manifolds	162
§7.2.	An abstract version of the Stable Manifold Theorem	166
§7.3.	Basic properties of stable and unstable manifolds	176
Chapter	8. Absolute Continuity of Local Manifolds	185
§8.1.	Absolute continuity of the holonomy map	187
§8.2.	A proof of the Absolute Continuity Theorem	195
§8.3.	Computing the Jacobian of the holonomy map	201
§8.4.	An invariant foliation that is not absolutely continuous	203
Chapter	9. Ergodic Properties of Smooth Hyperbolic Measures	205
§9.1.	Ergodicity of smooth hyperbolic measures	205
§9.2.	Local ergodicity	214
§9.3.	The entropy formula	220

Chapter 10. Geodesic Flows on Surfaces of Nonpositive Curvature	235
§10.1. Preliminary information from Riemannian geometry	236
§10.2. Definition and local properties of geodesic flows	238
§10.3. Hyperbolic properties and Lyapunov exponents	240
§10.4. Ergodic properties	246
§10.5. The entropy formula for geodesic flows	251
Chapter 11. Topological and Ergodic Properties of General Hyperbolic Measures	с 255
§11.1. Hyperbolic measures with local product structure	256
§11.2. Periodic orbits and approximations by horseshoes	261
§11.3. Shadowing and Markov partitions	262
Part 2. Selected Advanced Topics	
Chapter 12. Cone Technics	269
§12.1. Introduction	269
§12.2. Lyapunov functions	271
§12.3. Cocycles with values in the symplectic group	275
Chapter 13. Partially Hyperbolic Diffeomorphisms with Nonzero Exponents	277
§13.1. Partial hyperbolicity	278
§13.2. Systems with negative central exponents	281
§13.3. Foliations that are not absolutely continuous	283
Chapter 14. More Examples of Dynamical Systems with Nonzero Lyapunov Exponents	289
§14.1. Hyperbolic diffeomorphisms with countably many ergodic components	289
§14.2. The Shub–Wilkinson map	299
Chapter 15. Anosov Rigidity	301
§15.1. The Anosov rigidity phenomenon. I	301
§15.2. The Anosov rigidity phenomenon. II	309
Chapter 16. C^1 Pathological Behavior: Pugh's Example	315
Bibliography	321
Index	327